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Application
of Transversal Equalizer
to Radar Receiver
with Emphasis
on Doppler Sensitivity

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LINCOLN LABORATORY

APPLICATION OF TRANSVERSAL EQUALIZER
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WITH EMPHASIS ON DOPPLER SENSITIVITY

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ABSTRACT

There is a basic constraint imposed on radar measurements when the transmitted signal is a simple, CW pulse. As a result, the radar designer may wish to use a more complex signal and thereby extract more information from an echoing object. Unfortunately, the processed returns of the sophisticated waveforms usually consist of a peak response and small adjacent responses (called "time sidelobes") which degrade the resolving capability of the radar.

One means of reducing the sidelobes is to use a transversal equalizer in the radar receiver. A mathematical model of the equalizer is presented in this article and the sensitivities of an equalized system to uncompensated Doppler frequency shifts are computed. It is shown that the Doppler frequency band over which the equalizer is effective decreases as (1) the signal bandwidth decreases, (2) the distances of the large sidelobes from the main pulse increase, and (3) the magnitudes of the uncompensated sidelobes increase.

Testing the equalizer in a highly sensitive, tracking radar indicates that sidelobes 40 db below the main response can be obtained for a fixed Doppler frequency shift. Experimental data illustrating the stability of an equalized system in the presence of a Doppler shift show that, for a typical pulse-compression system, the equalizer utility is generally confined to situations where the frequency shift is within ± 2 percent of the signal bandwidth.

CONTENTS

APPLICATION OF TRANSVERSAL EQUALIZER
TO RADAR RECEIVER
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I. INTRODUCTION

It is desirable for a radar signal to have a long duration, and thereby a large energy content, so that one enjoys a high signal-to-noise ratio when observing the return from an object. Also, it is often required that the bandwidth of the signal be large so that the radar can easily resolve objects when more than one is present and so that the accuracy with which the range to an object is measured will be high.[1] Thus, a fundamental constraint exists when a rectangular-envelope CW pulse is used as a radar waveform, since the pulse bandwidth is inversely related to the pulse duration and one can be increased only at the expense of the other. Therefore, one must either operate with less than ideal parameters or employ a more sophisticated signal.

For some time now, work has progressed on the design of radar signals that have both long durations and large bandwidths (termed "large time-bandwidth product signals"). Dicke,[2] Darlington,[3] and Cauer[4] have been issued patents on a linear frequency-modulated pulse-compression waveform.* Work on the design of nonlinear FM pulse-compression signals is documented by Key, et al.,[8] and Fowle.[9] Phase-reversal signals represent another type of large time-bandwidth signals and are reported by Barker,[10] and DeLong.[11] In addition, many others have contributed a great deal toward the evolution of various types of coded radar waveforms.†

Associated with the techniques of maintaining long durations and large bandwidths are several characteristics which may prove troublesome. Perhaps the most severe of these (and the one with which this report is concerned) is the presence of time (range) sidelobes (small responses adjacent to the main object return) accompanying the peak response at the radar receiver output. These small neighboring lobes associated with the return from a large target can mask the return from a smaller, nearby object, thus reducing the effective dynamic range of the radar and severely degrading the ability of the radar to resolve several objects differing in size but located near one another in range. This limitation is so serious that much emphasis is placed on the ability to keep the values of these sidelobes at tolerable levels.

There are a number of factors that contribute to the magnitudes of these sidelobes. One of the prime sources is the nonideal transfer characteristics of the networks in the signal flow chain. Since pulse coding and decoding networks (pulse expansion and compression filters) are generally quite complex in nature, it is quite likely that their transfer characteristics may deviate considerably from that which is desired.‡ Another prime source is the residual amplitude and phase

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* Various linear FM waveforms are described by Klauder, et al.,[5] Cook,[6] and Fowle, et al.[7]

† An excellent analysis and description of a large number of radar waveforms can be found in Fowle.[12]

‡ For example, see O'Meara[13] for a discussion on the construction of passive circuits for pulse expansion and compression networks.

perturbations inherent in the radar signal design. The most notable of these is the presence of Fresnel ripple in the spectra of rectangular-envelope, frequency-modulated waveforms.[*] Each of these two causes has the net effect of placing unwanted distortions in the spectra of the receiver output signals.

Inasmuch as sidelobes resulting from distortions can be viewed as combinations of attenuated replicas of the main pulse displaced about the peak response,[†] one can conceive of their removal by delaying (or advancing), attenuating, and subtracting the peak response from the sidelobes. A device that does just this is termed a "transversal equalizer" and has been employed for many years for reducing distortions in radio and television transmitting equipment.[14-17] Holsinger[18] describes the use of an equalizer to compensate for phase distortions in dispersive telephone lines. Also, Key, et al.,[19] describe how a transversal equalizer is used to reduce the sidelobes accompanying the matched radar receiver[‡] output of a phase-reversal signal, and Pratt[21] reports on some aspects of an equalizer for use with a large-bandwidth radar signal.

Of particular concern is the sensitivity of the (transversal) equalized sidelobes to a Doppler frequency shift in the returning radar echo. As will be shown later, the sensitivity depends upon (among other things) the magnitude of the signal bandwidth, the radar carrier frequency, and the velocity of the observed object. There are presently a number of radars whose parameters and targets of interest are such that the expected value of Doppler frequency shift is an important consideration when judging the possible usefulness of a transversal equalizer.

The next section briefly reviews the effects of signal distortion. A mathematical model of a transversal equalizer is presented and analytical results of equalizing sidelobes resulting from both amplitude and phase distortions are given. Particular emphasis is then placed on the performance of an equalized system to a Doppler frequency shift. Experimental results were obtained by placing a prototype equalizer in a radar receiver and tracking several objects. Data on the output waveforms are presented when a UHF and an L-band radar view both stationary and moving (satellites) targets.

II. ANALYSIS

In this section, a mathematical model of the transversal equalizer is given and special equalizer settings are considered for the reduction of sidelobes which result first, from amplitude distortions and then, from phase distortions. After that, the sensitivity of the equalizer to a Doppler frequency shift will be derived and the results plotted as a set of curves. First, however, let us lay the groundwork for further analysis by briefly reviewing the effects of amplitude and phase distortions on a signal from the standpoint of "paired echoes."[§]

A. Paired Echoes

For the sake of convenience, consider only the positive frequency portions of the signals and passband spectra. Also assume that the spectra are narrowband and therefore bounded.
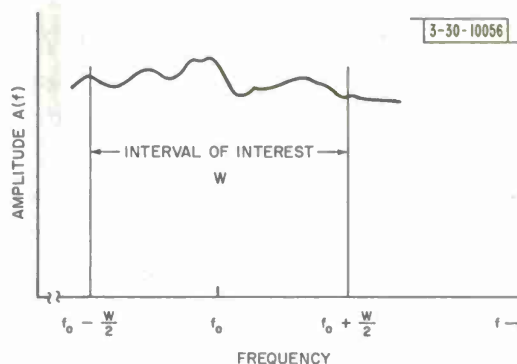
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[*] Klauder, et al.,[5] discuss the presence of Fresnel ripple on linear FM signals.

[†] This notion of sidelobes is discussed in the following section.

[‡] The matched receiver is discussed by Woodward.[20]

[§] For further discussions on paired echo distortions, see Refs. 5 and 22.

Fig. 1. Distortion in spectral modulus of voltage transfer characteristic.

Let $F(f) = A(f) \exp[j\Theta(f)]$, where $F(f)$ is the voltage transfer function of a unit (filter, amplifier, etc.) in the signal flow path and where $A(f)$ and $\Theta(f)$ are the modulus and phase portions, respectively.

The term $A(f)$ may appear as the curve in Fig. 1 over the interval of interest and can be represented as the expansion

$$A(f) = 1 + 2 \sum_{n=1}^{\infty} a_n \cos 2\pi \, \frac{n}{W} \, (f - f_o) + 2 \sum_{n=1}^{\infty} b_n \sin 2\pi \, \frac{n}{W} \, (f - f_o) \tag{1}$$

in the interval from

$$(f_o - \frac{W}{2}) \leqslant f \leqslant (f_o + \frac{W}{2})$$

and zero elsewhere. The symbols a_n and b_n are the usual Fourier coefficients, W is the width of the band, and the over-all gain is normalized. Furthermore,

$$A(f) = 1 + 2 \sum_{n=1}^{\infty} X_n \cos 2\pi \, [\frac{n}{W} \, (f - f_o) - \varphi_n] \tag{2}$$

where

$$X_n = (a_n^2 + b_n^2)^{1/2}$$

and

$$2\pi\varphi_n = \arctan \frac{b_n}{a_n} \quad .$$

Let $s(t) = g(t) \cos 2\pi f_o t$ be a signal with a positive frequency spectrum given by $S(f) \exp[j\eta \, (f)]$ in the interval W and zero elsewhere. When the signal is passed through the network with a modulus given by Eq. (2) (and linear phase), the output $s_o(t)$ is given by

$$s_o(t) = s(t) + \sum_{n=1}^{\infty} X_n \, [g(t + \frac{n}{W}) \cos 2\pi \, (f_o t - \varphi_n) + g(t - \frac{n}{W}) \cos 2\pi \, (f_o t + \varphi_n)] \tag{3}$$

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where the constant time delay term required for physical realizability has been omitted.

Hence, the signal is accompanied by delayed (and advanced), attenuated replicas of the input signal. The amount of delay is determined (in pulse widths) by n, the number of cycles of ripple across the passband.

In a similar manner, let $A(f)$ be flat over the band but let $\Theta(f)$ equal

$$\frac{c_o}{2} + \sum_{n=1}^{\infty} \cos 2\pi \frac{n}{W} (f - f_o) + \sum_{n=1}^{\infty} d_n \sin 2\pi \frac{n}{W} (f - f_o) \quad . \tag{4}$$

Now,

$$e^{j\Theta(f)} = \prod_{n=1}^{\infty} \left[\sum_{k=-\infty}^{\infty} (j)^k J_k(c_n) e^{jk2\pi \frac{n}{W} (f-f_o)} \right] \left[\sum_{\ell=-\infty}^{\infty} J_\ell(d_n) e^{j\ell 2\pi \frac{n}{W} (f-f_o)} \right]$$

where $J_n(x)$ represents the Bessel function of the first kind. For convenience, c_o has been set to zero. If c_n, $d_n \ll 1$, then $J_k(c_n)$ and $J_\ell(d_n)$ can be set equal to zero for k or $\ell \geqslant 2$. Also,

$$J_1(c_n) \approx J_1(d_n) \approx 0$$

and

$$J_o(c_n) \approx J_o(d_n) \approx 1 \quad .$$

Then the resulting function $P(f)$ is given by

$$P(f) = \prod_{n=1}^{\infty} \left[1 + Y_n \left(e^{j2\pi[\frac{n}{W} (f-f_o)+\alpha_n]} - e^{-j2\pi[\frac{n}{W} (f-f_o)+\alpha_n]} \right) \right] \tag{5}$$

where

$$Y_n = [-J_1^2(c_n) + J_1^2(d_n)]^{1/2}$$

and

$$2\pi\alpha_n = \arctan \frac{J_1(c_n)}{J_1(d_n)} \quad .$$

Since $Y_n \ll 1$, we neglect terms involving products of the Y_n's. Finally, the normalized value is

$$P(f) = 1 + j2 \sum_{n=1}^{\infty} Y_n \sin 2\pi[\frac{n}{W} (f - f_o) + \alpha_n] \quad . \tag{6}$$

When the signal $s(t)$ is fed through the network given by Eq. (6), the output is

$$s_o(t) = s(t) + \sum_{n=1}^{\infty} Y_n[g(t + \frac{n}{W}) \cos 2\pi(f_o t + \alpha_n) - g(t - \frac{n}{W}) \cos 2\pi(f_o t - \alpha_n)] \quad . \tag{7}$$
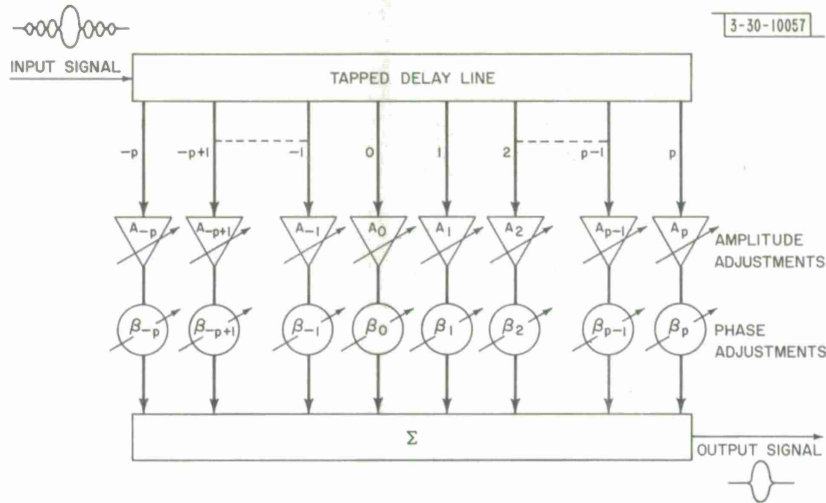
4

Fig. 2. Transversal equalizer. A_n's and β_n's represent amplitude and phase adjustments, respectively.

This result is similar to that in Eq. (3) for amplitude distortion, except that the advanced echo has an opposite polarity to that of the delayed replica.

B. Transfer Characteristic of a Transversal Equalizer

Consider a transversal equalizer with $2p + 1$ taps spaced τ seconds apart. As shown in Fig. 2, each tap has a provision for adjusting both the gain and phase of the signal. The impulse response is then

$$h(t) = \sum_{n=-p}^{p} A_n \delta(t - T - n\tau)\, e^{+j2\pi\beta_n} \tag{8}$$

where A_n and β_n represent the gain and phase settings, respectively, at the nth tap. The spectrum of this function is

$$H(f) = \int_{-\infty}^{\infty} \sum_{n=-p}^{p} A_n \delta(t - T - n\tau)\, e^{+j2\pi\beta_n}\, e^{-j2\pi ft}\, dt \tag{9}$$

Interchanging the order of integration and summation, evaluating, and considering only the positive frequency portion of the spectrum referenced to f_o gives

$$H(f) = e^{-j2\pi(f-f_o)T} \sum_{n=-p}^{p} A_n\, e^{-j2\pi n\tau(f-f_o)}\, e^{+j2\pi\beta_n} \tag{10}$$

which represents the equalizer voltage transfer characteristic.

C. Amplitude Compensation

Let us, for convenience, assume that the center tap of the equalizer has zero attenuation and phase shift ($A_o = 1$, $\beta_o = 0$). If we adjust the remaining gains and phases such that $A_n = A_{-n}$ and $\beta_n = \beta_{-n}$, Eq. (10) can be written as

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$$H(f) = 1 + 2 \sum_{n=1}^{p} A_n \cos 2\pi[n\tau(f - f_o) - \beta_n] \qquad (11)$$

where T, which must be equal to or greater than $p\tau$ for physical realizability, is chosen to be equal to zero. Equation (11) is the expression for a bandpass filter with a linear phase characteristic. Consequently, it can be used to compensate for amplitude distortions without introducing phase distortions.

If we let $\tau = 1/W$, $\beta_n = \varphi_n$ and $A_n = -X_n$ in Eq. (11) and cascade this unit with one whose general amplitude characteristic is given by Eq. (2), we have (if we consider the first p terms)

$$A(f)\, H(f) = 1 - 2 \sum_{n=1}^{p} \sum_{m=1}^{p} X_n X_m \left\{ \cos 2\pi[\frac{n+m}{W}(f-f_o) - (\varphi_n + \varphi_m)] \right.$$

$$\left. + \cos 2\pi[\frac{n-m}{W}(f-f_o) - (\varphi_n - \varphi_m)] \right\} \qquad . \qquad (12)$$

The original distortion lobes of magnitude X_n have been replaced by a multitude of distortions of value $X_n X_m$. Since X is generally much less than unity, these small distortions will yield very small sidelobes which will, in most practical cases, be an order of magnitude smaller than the size of the sidelobes given in Eq. (3).
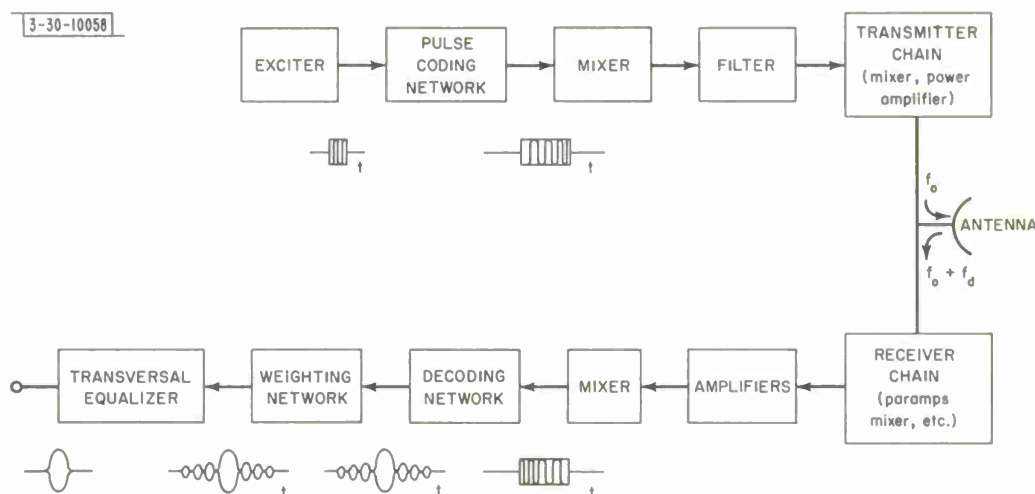

Fig. 3. Simplified block diagram of radar employing coded waveform.

A special problem exists when the transversal equalizer is aligned to compensate for a particular network and a signal is passed through that network and is then shifted in frequency before it is fed into the equalizer. This is the case when the equalizer is placed in a radar receiver and aligned to correct for distortions in the radar exciter and transmitter (Fig. 3). The waveform, after passing through a portion of the signal path may be shifted in frequency (as a result of the well-known Doppler effect)* before entering the receiver. If the magnitude of the

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* The spectrum of a narrow-band signal that is reflected from a moving object is translated in frequency by an amount known as the Doppler frequency shift. The magnitude of this shift is equal to $(2v/c)f_o$ where v is the object's relative velocity, f_o is the radar carrier frequency, and c is the velocity of light.

frequency shift is unknown, or if it can vary, it becomes difficult to equalize the receiver for all operating conditions. Consequently, there is a need to understand the effect of such a frequency shift on the sidelobe levels.

Suppose that $A(f)$ is separated from $H(f)$ by an unknown Doppler shift f_d. If $H(f)$ is aligned when f_d is equal to zero, the cascaded transfer function is given as

$$A(f + f_d)\, H(f) \approx 1 + 2 \sum_{n=1}^{p} X_n \left\{ \cos 2\pi\left[\frac{n}{W}(f - f_o + f_d) - \varphi_n\right] \right.$$

$$\left. - \cos 2\pi\left[\frac{n}{W}(f - f_o) - \varphi_n\right] \right\} \quad . \tag{13}$$

The third-order (double sum) terms have been neglected, since the effect of f_d is not to increase the magnitude of the sidelobes resulting from them. Expression (13) reduces to

$$A(f + f_d)\, H(f) = 1 + 4 \sum_{n=1}^{p} X_n \left\{ \cos 2\pi\left[\frac{n}{W}\left(f - f_o + \frac{f_d}{2}\right)\right.\right.$$

$$\left.\left. - \varphi_n + 1/4\right]\right\} \sin \pi \frac{n}{W} f_d \quad . \tag{14}$$

The output signal can be found by multiplying Eq. (14) by $S(f + f_d)\, \exp\,[j\Theta(f + f_d)]$ and taking the transform in a similar manner as before. It is not difficult to see that the magnitudes of the distortion sidelobes are given by $2X_n \sin \pi (n/W)\, f_d$. It may not be too surprising to have found that the time sidelobe magnitudes increase as:

(1) The signal bandwidth decreases,
(2) The Doppler frequency shift increases,
(3) The magnitudes of the uncompensated sidelobes increase,
(4) The displacements between the sidelobes and main pulse increase.

If the lobes are not perfectly nulled for zero frequency shift, the resulting residual side-lobe values will reflect this imperfect cancellation. However, for small n and moderate values of Doppler shift, this contribution is small and is therefore not considered.

Figure 4 is a plot of the magnitudes of sidelobes which result when a Doppler shift is injected between the source of the distortion and the equalizer. The Doppler frequency shift is given as a percent of the signal bandwidth and n, the harmonic of the ripple (equal to the reciprocal of the distance that the sidelobe is displaced), is a parameter. It is assumed that perfect cancellation exists when f_d equals zero.

D. Phase Compensation

Adjusting the equalizer gains and phases such that $A_n = -A_{-n}$ and $\beta_n = -\beta_{-n}$ gives a voltage transfer function of

$$H(f) = 1 - 2j \sum_{n=1}^{p} A_n \sin 2\pi[n\tau(f - f_o) - \beta_n] \quad . \tag{15}$$

Letting $\tau = 1/W$, $\beta_n = -\alpha_n$, and $A_n = Y_n$ and cascading with the first p terms of the phase characteristic in Eq. (6), we have
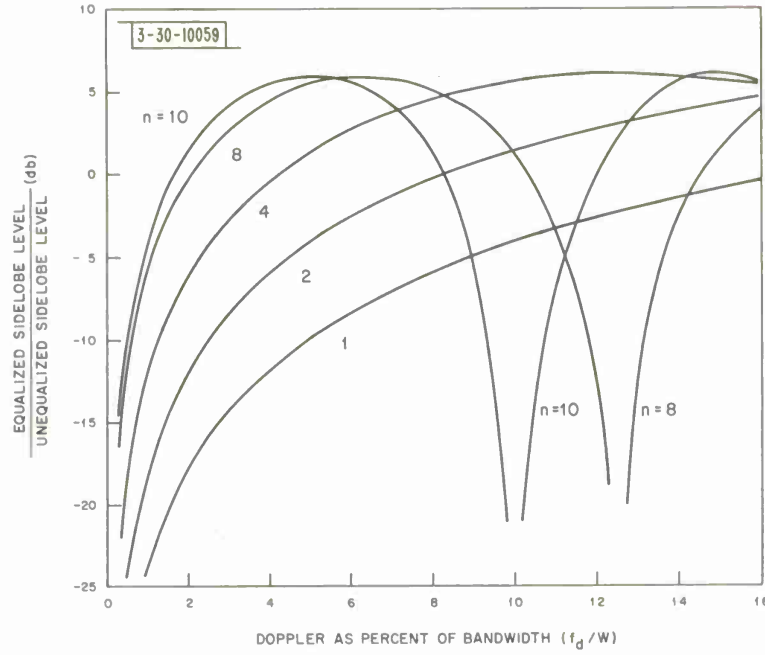
Fig. 4. Plot of normalized sidelobe values vs Doppler frequency shift.
Distance of sidelobe from peak pulse is n, measured in pulse widths.

$$P(f)\, H(f) = 1 + 2 \sum_{n=1}^{p} \sum_{m=1}^{p} Y_n\, Y_m \{\cos 2\pi[\frac{n-m}{W}\,(f - f_o) + (\alpha_n - \alpha_m)]$$

$$- \cos 2\pi[\frac{n+m}{W}\,(f - f_o) + (\alpha_n + \alpha_m)]\} \quad . \tag{16}$$

The transfer function $P(f)$ is that of an all-pass device with phase perturbations.   When cascaded with the equalizer such that the first-order sidelobes are canceled (recall that the lower-order sidelobe terms were discarded), a small amplitude ripple is introduced which yields many sidelobes with their magnitudes given by $Y_n\, Y_m$. Again, the magnitudes of these lobes are small and they will be ignored in the remainder of this article.

As before, consider the effect of a Doppler frequency shift introduced in the system between the distortion source and the compensation.   The transfer function becomes

$$P(f + f_d)\, H(f) = 1 + 4j \sum_{n=1}^{p} Y_n \left\{ \sin 2\pi \left[\frac{n}{W} \left(f - f_o + \frac{f_d}{2} \right) \right. \right.$$

$$\left. \left. + \alpha_n + 1/4 \right] \right\} \sin \pi\, \frac{n}{W}\, f_d \quad . \tag{17}$$

When the signal is considered along with the above expression, the increase in sidelobe level due to a Doppler frequency shift is seen to be $2Y_n \sin \pi(n/W)\, f_d$ which is similar to the result obtained for the amplitude case.   Hence, the previous comments of Sec. II-C, as well as the curves of Fig. 4, apply for the equalization of sidelobes created by either amplitude or phase distortions.

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III. SYSTEM DESCRIPTION

A breadboard transversal equalizer was designed and constructed* and then tested in an existing radar system. First, the configuration of the equalizer will be discussed, followed by a brief description of the radar.

A. Transversal Equalizer

Figure 5 is a block diagram of the equalizer. The input is an IF pulse centered at a carrier frequency of 1.5 Mcps (plus Doppler) and with a duration slightly greater than 1 μsec between the −3-db points. A total of eleven taps, spaced at 1-μsec intervals, permit an overall equalization interval of 10 μsec about the peak pulse. The outputs of each tap are adjusted for gain and phase, summed, and the resultant signal is fed into the remainder of the radar system.

The interval of compensation can be placed in any 10-μsec segment about the pulse center; for example, Fig. 5 gives the connection when the region of equalization is 1 μsec before the peak to 9 μsec following the pulse. The 1-μsec delays are obtained by the placement of coaxial delay cables between taps and the losses in each section of cable are offset by the gains of the transistorized buffer amplifiers.

Each parallel channel (except that of the main peak channel) has its own amplifier, gain adjustment, and phase adjustment. The gain of the peak channel is normally set 20 to 35 db above that of the remaining tapped channels.

B. Radar System

The radar to which the equalizer was connected for tests is a highly sensitive, precision tracking radar known as TRADEX† and is located in the Kwajalein Atoll. An object located at a distance of up to 1500 nautical miles with a backscattering cross section of one square meter can be tracked in azimuth, elevation, range, and Doppler, and the pulse returns from both UHF (425 Mcps) and L-band (1320 Mcps) carrier frequencies can be viewed simultaneously. Hence, the radar is well suited for tracking satellites. Although the system uses a variety of signals, only the linear FM waveform was employed for the tests and, therefore, only the equipment associated with this pulse-compression mode of operation will be described.

A block diagram showing the signal generation and flow path is given in Fig. 6. A short pulse is fed into a passive expansion filter (constructed of bridged-tee, all-pass networks) that modulates the pulse so that its duration is 50 μsec and its carrier frequency sweeps through a 1-Mcps band in a nearly linear fashion. The pulse is then heterodyned to 60 Mcps, limited, gated, mixed to the proper carrier frequency (425 or 1320 Mcps), amplified, and transmitted.

Upon reception, the waveform is amplified in a low-noise device, mixed to a carrier frequency of 1.5 Mcps, and then compressed. Prior to weighting the signal to reduce those time sidelobes associated with the signal design, the Doppler frequency is removed by mixing the return echo with an oscillator whose frequency is derived from a Doppler tracking servomechanism. As a result, the signal spectrum of the tracked return is always centered in the passband of the weighting filters; consequently, the effectiveness of the weighting filter in reducing these lobes is not dependent upon the value of Doppler frequency.

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* The unit was designed and built by the Radio Corporation of America, Moorestown, N. J.

† A considerably more detailed description of TRADEX, as it existed when the measurements were obtained, can be found in Ref. 23.

Fig. 5.   Transversal equalizer.   Configuration shown is for equalized interval 1 μsec before to 9 μsec following peak response.
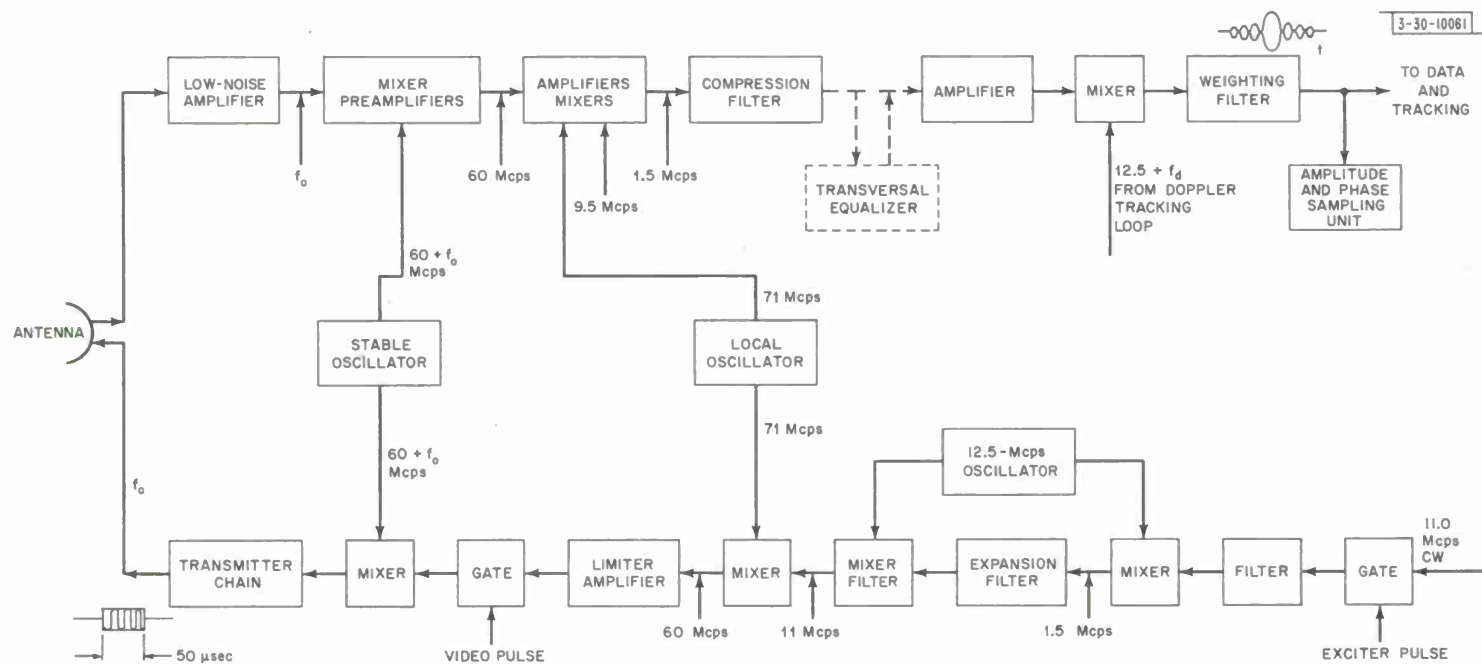
Fig. 6. Radar used in testing transversal equalizer. Carrier frequency (f₀) is 425 Mcps for UHF radar and 1320 Mcps for L-band. Configuration of radar has been extensively changed since time at which tests were conducted.

3-30-10062
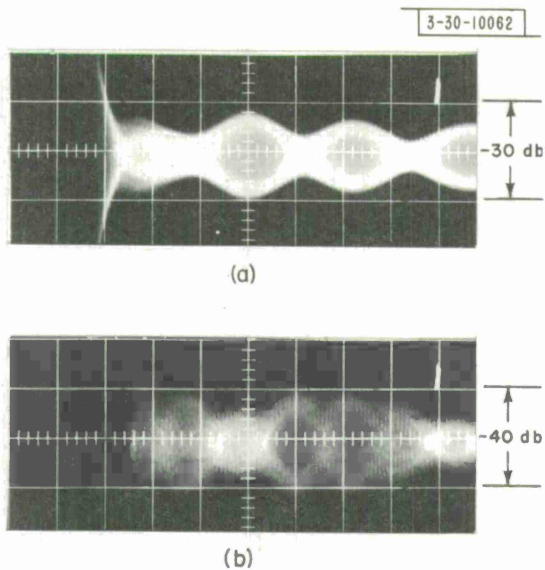
−30 db

(a)

−40 db

(b)

Fig. 7. Photographs of returns from 6-inch sphere viewed at receiver output. (a) Transversal equalizer not inserted. Two large divisions represent a magnitude 30 db below peak value. (b) Transversal equalizer is aligned in receiver. Two large divisions represent a magnitude of −40 db.

Fig. 8. Returns from 6-inch sphere with and without equalizer connected. Magnitudes were sampled at 1-µsec intervals and each point is average of several thousand pulse returns.
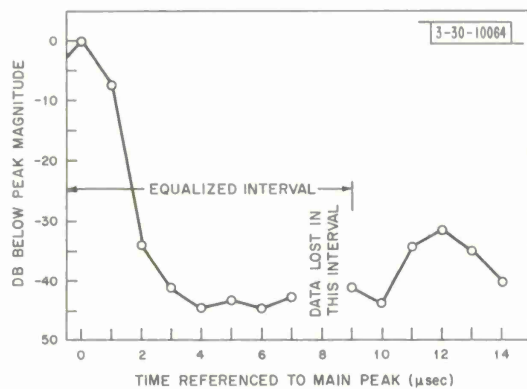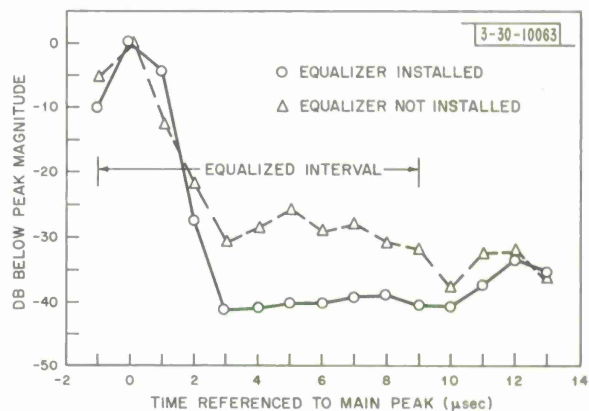

3-30-10063

O EQUALIZER INSTALLED

△ EQUALIZER NOT INSTALLED

EQUALIZED INTERVAL

DB BELOW PEAK MAGNITUDE

TIME REFERENCED TO MAIN PEAK (µsec)

3-30-10064

EQUALIZED INTERVAL

DATA LOST IN THIS INTERVAL

DB BELOW PEAK MAGNITUDE

TIME REFERENCED TO MAIN PEAK (µsec)

Fig. 9. Return from Echo I viewed at UHF when relative Doppler shift was zero.

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One of the prime radar data devices is a unit that samples the values of the amplitude (and phase) from a single-pulse return at intervals separated by 1 μsec. The amplitude of the waveform is measured by passing the signal through a logarithmic detector, sampling and digitizing the result, and recording the output on magnetic tape. The parameters are adjusted so that the amplitude number represents relative signal strength in decibels with the least significant bit representing 1 db.

For the tests described in the next section, the data sampling intervals were arranged so that 16 contiguous samples (separated in time by 1 μsec) were taken of the amplitudes of each pulse return. The initial measurement was made 1 μsec prior to the time at which the peak value arrived and the last sample was taken 15 μsec later. Thus, the magnitudes of the pulse sidelobes could be determined relative to the main pulse in the interval immediately following the peak return.

IV. TEST RESULTS

The equalizer was inserted in the radar receiver directly following the pulse-compression filter (Fig. 6) but prior to the point where the Doppler frequency is removed from the signal return. Therefore, when alignment is made to a signal return with a given Doppler frequency shift, the increases in sidelobe values, as a function of a change in Doppler shift, depend upon the magnitudes of the distortions in the exciter and transmitter chains. Sidelobes that result from receiver distortions in the frequency band occupied by the zero Doppler signal will be insensitive to Doppler shift.

A 6-inch balloon-borne sphere was tracked and the sidelobes accompanying the main return were observed with: (1) the receiver in its normal configuration, and (2) the transversal equalizer inserted in the UHF receiver and aligned for low sidelobes in the interval from 1 μsec leading to 9 μsec trailing the peak. Figure 7(a) shows the unequalized return. The main peak is located at the left edge and each large horizontal division represents one microsecond. The vertical display is linear and the scale is adjusted so that a sidelobe occupying two large divisions (peak to peak) is 30 db below the main value. Figure 7(b), taken with the equalizer connected and aligned, has the same horizontal time scale; however, the vertical scale is such that two major divisions now represent −40 db. Notice that the reduction in sidelobe values is about 10 db.

Quantitatively, the results can be easily viewed by sampling the magnitudes at 1-μsec intervals as described in Sec. III, averaging the samples[*] for many pulse returns, and plotting the final values. The results determined by this means for the two cases discussed above are shown in Fig. 8 with the individual data points connected by straight lines.

A meaningful test to determine the response of the equalized system to a Doppler frequency is to track a satellite as it makes the overhead pass. The return signal will thereby exhibit a continually changing Doppler shift during the transit. This was done with both the UHF and L-band radar systems.

With the equalizer aligned in the UHF receiver, Echo I was acquired when the Doppler shift was +15 kcps (equivalent to a radial velocity of 17,500 feet/sec) and was tracked until the Doppler shift became −15 kcps. The receiver output waveform was sampled at various times during the pass. Figure 9 gives a plot of the waveshape when the Doppler frequency was zero. As in the case of the sphere return, the sidelobes are at least 40 db below the peak response in the equalized interval.

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* The logarithmic amplitude numbers are converted to linear values (representing powers), averaged, and the result is returned to logarithmic form (using $10 \log_{10} X$).
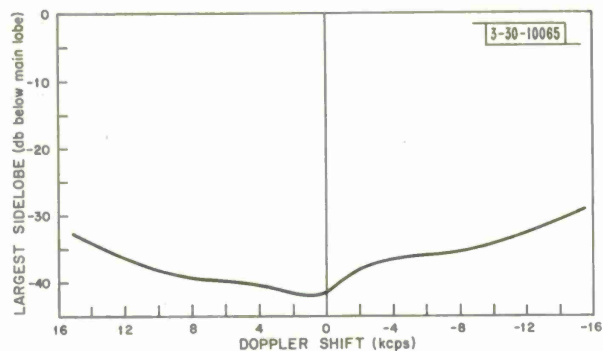
Fig. 10. Largest sidelobe (in equalized interval) as a function of Doppler shift for returns from Echo I viewed at UHF.
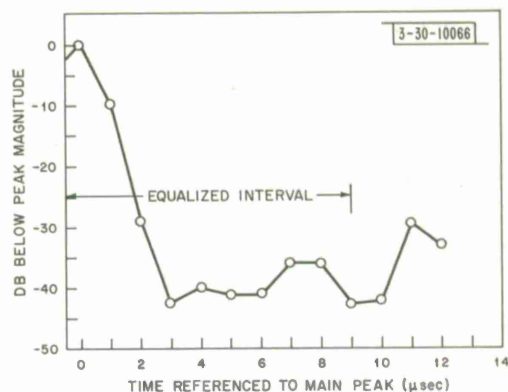

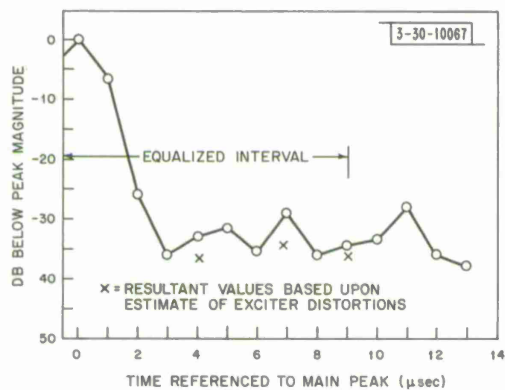Fig. 11. Return from Echo I viewed at L-band when relative Doppler shift was zero.

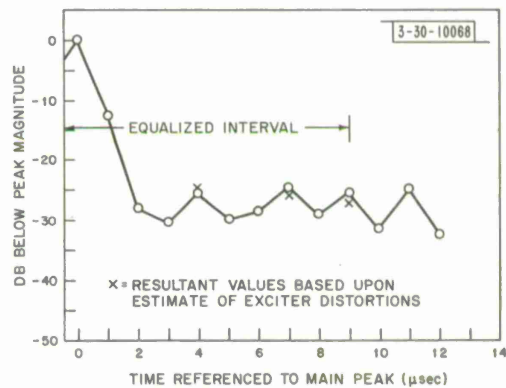Fig. 12. Echo I return viewed at L-band when relative Doppler shift was −10 kcps.

Fig. 13. Echo I return viewed at L-band when relative Doppler shift was −40 kcps.

A plot of the magnitude of the largest lobe as a function of the frequency shift is given in Fig. 10. Notice that the largest sidelobe (again, our interest is confined to the equalized interval) is 30 db below the peak and occurs when the Doppler frequency is −15 kcps. Although Eqs. (14) and (17) show that the sidelobe magnitudes are independent of the sign of the shift, the experimentally derived curve is not an even function about the zero Doppler line. This lack of symmetry could be caused by imperfect phase adjustments, measurement errors including amplitude quantization, and unsymmetrical receiver distortions located immediately outside the frequency band occupied by the zero Doppler signal.

At a later date, Echo I was again tracked under similar circumstances but with the equalizer aligned in the L-band receiver. Since the shift in Doppler is more pronounced at this higher radar carrier frequency, a better indication of the sidelobe sensitivity as a function of frequency shift was obtained.

A plot of the wave shape of the satellite return (viewed at the receiver output) when the relative velocity was zero is given in Fig. 11. Notice that the sidelobes are about 40 db below the main peak with the exception of one lobe with a relative magnitude of −36 db. The presence of this larger sidelobe may have been caused by a drift in the system alignment.[*]

A considerable increase in the sidelobe level is apparent when the return echo is shifted by −10 kcps as illustrated in Fig. 12. The magnitude of each lobe has increased in value by about 7 db. When the Doppler is shifted by a greater amount a larger degradation is observed. Figure 13 plots the waveform when the shift is −40 kcps and the sidelobes are only 25 db below the main response.

Observe that, as a function of Doppler shift, the sidelobe increases are nearly uniform across the 9-μsec interval and that no one sidelobe has a magnitude that is predominant. This point should be considered when viewing Fig. 14 which gives a plot of the largest sidelobe as a function of frequency. For this case, the largest sidelobe is 24 db below the peak response and occurs when the Doppler shift is −25 kcps. Again, the curve is slightly asymmetric as it was in the UHF results.

Fig. 14. Largest sidelobe as a function of Doppler shift for return from Echo I viewed at L-band.
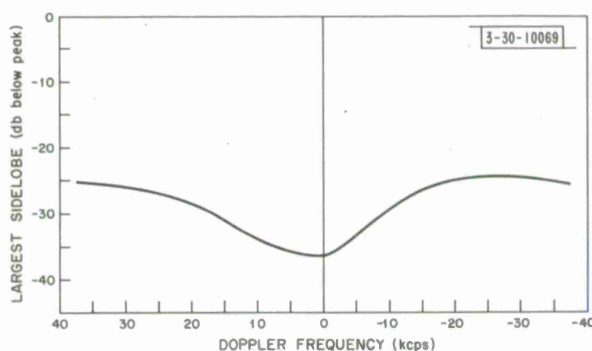
A careful examination of the satellite returns revealed that the magnitude of the pulse width as measured between the −3-db points was reduced by about 15 percent from previously measured values. Since the pair of sidelobes adjacent to the main pulse are usually contained within the envelope of this pulse (and are not always distinguished as sidelobes) they tend to broaden this main return. By proper alignment of the equalizer, these adjacent sidelobes are reduced and the pulse width is decreased.

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[*] Equalizer alignment was performed by using the return from a 6-inch sphere about one hour prior to the satellite track.

Since it is quite difficult to determine the exact magnitudes of the exciter distortions, precise comparisons between the analytical and experimental results cannot be easily made; however, some basic correlation between the results is evident. For example, it is believed that the sidelobes resulting from exciter distortions become smaller in value when their distances from the main pulse increase. Although the more distant lobes are increasingly sensitive to a small percentage Doppler shift, their unequalized values are initially lower; therefore, for small shifts, their size becomes comparable to the size of the initially larger, but less sensitive, near-in sidelobes. As noticed in Fig. 13, the values seem to be somewhat uniform in magnitude.

It is perhaps not unreasonable to assume that the exciter distortions for the L-band system are such that they are responsible for sidelobe values of about −25, −28, and −31 db at respective distances of 4, 7, and 9 μsec from the main pulse. (These magnitudes are slightly smaller than those of the sidelobes viewed at the L-band receiver output.) If the lobes are completely removed by use of a transversal equalizer positioned in the receiver and then the signal is Doppler-shifted by a known amount, the resulting sidelobe values can be determined by using data from the plots of Fig. 4. For a 10-kcps offset, the computed magnitudes for these three lobes are plotted along with the observed values in Fig. 12. One would expect the actual values to be larger, since the imperfect cancellation is of importance (but not considered) for this rather small (2 percent) shift. When the Doppler is increased to 40 kcps, the computed values compare quite favorably with those obtained experimentally. The results for this case are compared in Fig. 13.

The usefulness of a transversal equalizer depends, of course, upon the application of the radar, the basic parameters of the radar signal, and the sidelobe values present at the (unequalized) receiver output. For example, the device may be useful if a radar has an L-band carrier frequency, a signal bandwidth of 75 kcps, and the fastest object to be observed is an airplane. On the other hand, an L-band radar used to track ballistic missiles or satellites may require a signal bandwidth of about 3 Mcps (or more) for an equalizer to be of value.

V. CONCLUSIONS

We have mentioned a basic constraint imposed upon radar measurements when the radar signal is a simple pulse, and have also considered the undesirable presence of time sidelobes associated with many of the more complex radar waveforms. A transversal equalizer was shown to provide a means of reducing the magnitude of these sidelobes. A mathematical model of the equalizer was presented and an expression for the sensitivity of an equalized system to an uncompensated Doppler frequency shift was derived. When the equalizer is placed in a radar receiver, the increase in the sidelobes as a function of Doppler depends upon the magnitudes of the distortions present in the radar exciter as well as the signal bandwidth and the distance of the sidelobe from the main pulse.

Results of testing the equalizer in a tracking radar show that sidelobes 40 db below the main response can be obtained for a fixed Doppler frequency shift. Furthermore, experimental values compare favorably with the derived Doppler sensitivity for moderate and large percentages of frequency shifts. For small shifts, the residual, compensated sidelobe levels achieved in practice are of importance and must be considered.
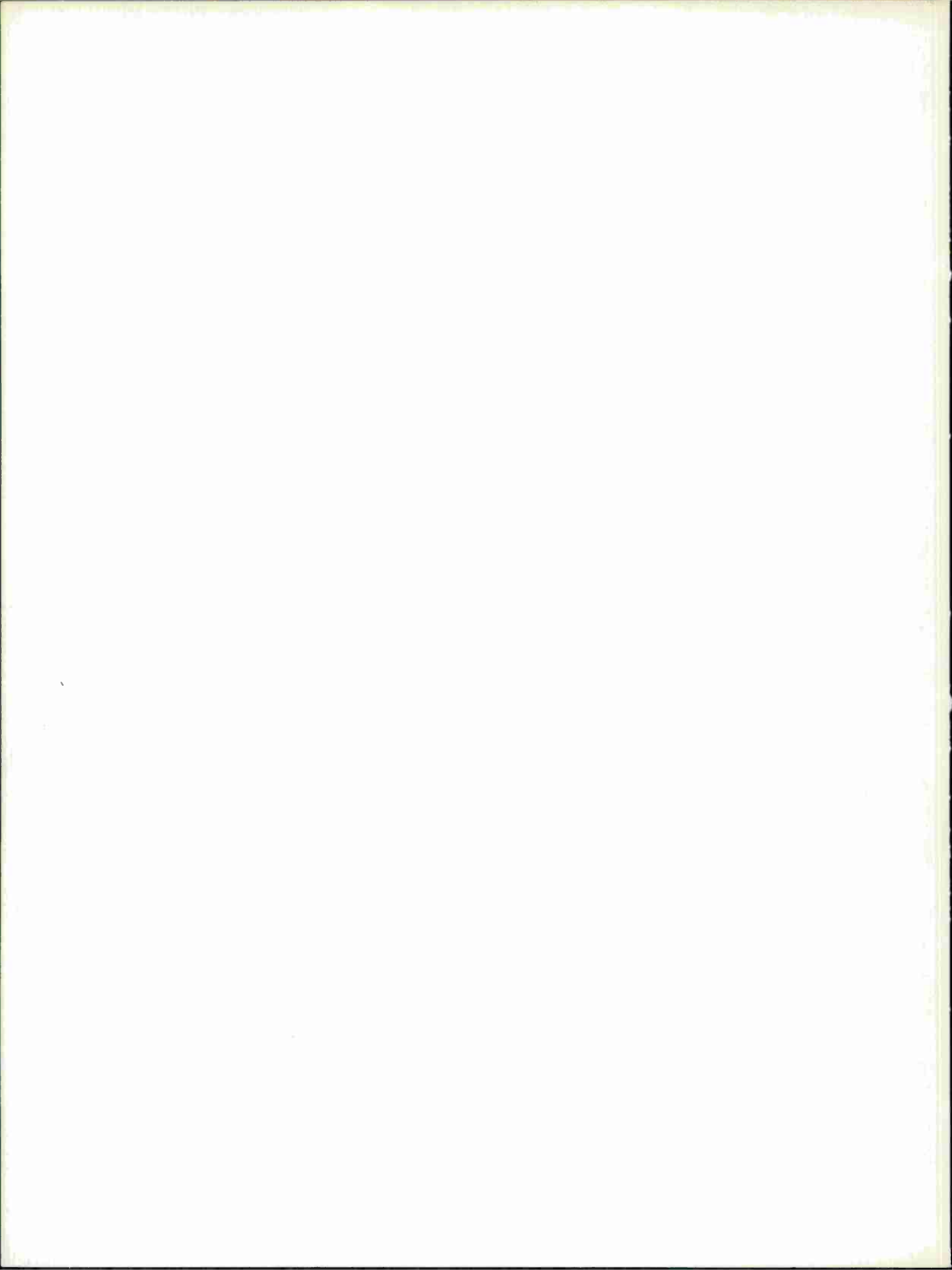
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REFERENCES

1. R. Manasse, "Summary of Maximum Theoretical Accuracy of Radar Measurements," MTS-2, MITRE Corporation, Bedford, Mass. (1 April 1960), not generally available.

2. R. H. Dicke, "Object Detection System," U. S. Patent No. 2,624,876, filed 14 September 1945.

3. S. Darlington, "Pulse Transmission," U.S. Patent No. 2,678,997, filed 31 December 1949.

4. W. A. Cauer, German Federal Republic, Patent No. 892,772, 19 December 1950.

5. J. R. Klauder, A. C. Price, S. Darlington and W. J. Albersheim, "The Theory and Design of Chirp Radars," Bell System Tech. J. $\underline{39}$, 745 (1960).

6. C. E. Cook, "Pulse Compression — Key to More Efficient Radar Transmission," Proc. IRE $\underline{48}$, 310 (1960).

7. E. N. Fowle, D. R. Carey, R. E. VanderSchuur and R. C. Yost, "A Pulse Compression System Employing a Linear FM Gaussian Signal," Proc. IEEE $\underline{51}$, 304 (1963).

8. E. L. Key, E. N. Fowle and R. D. Haggarty, "A Method of Pulse Compression Employing Nonlinear Frequency Modulation," Technical Report 207, Lincoln Laboratory, M.I.T. (13 August 1959), DDC 312903.

9. E. N. Fowle, "The Design of FM Pulse Compression Signals," IEEE Trans. on Inform. Theory $\underline{IT-10}$, No. 1 (1964).

10. R. H. Barker, "Group Synchronizing of Binary Digital Systems," Communications Theory, Willis Jackson, Ed. (Academic Press, London, 1958), pp. 273-287.

11. D. De Long, "Three-phase Codes," Group Report 47-28, Lincoln Laboratory, M.I.T. (24 July 1959), DDC 241519.

12. E. N. Fowle, "The Design of Radar Signals," MITRE Report SR-98 (1 November 1963), DDC 617711.

13. T. R. O'Meara, "The Synthesis of Band-Pass, All-Pass Time Delay Networks with Graphical Approximation Techniques," Hughes Aircraft Company, Research Report No. 114 (June 1959).

14. R. V. Sperry and D. Surenian, "A Transversal Equalizer for Television Circuits," Bell System Tech. J. $\underline{39}$, No. 2, 405 (1960).

15. B. C. Bellows and R. S. Graham, "Experimental Transversal Equalizer for TD-2 Radio Relay System," Bell System Tech. J. $\underline{36}$, 1429 (November 1957).

16. H. E. Kallmann, "Transversal Filters," Proc. IRE 28, 302 (July 1940).

17. J. M. Linke, "A Variable Time-Equalizer for Video-Frequency Waveform Correction," Proc. IRE 99, Part IIIa, 427 (1952).

18. J. L. Holsinger, "Digital Communications Over Fixed Time Continuous Channels with Memory — with Special Application to Telephone Channels," D. Sc. Thesis, Department of Electrical Engineering, M.I.T., Cambridge, Mass., October 1964.

19. E. L. Key, E. N. Fowle and R. D. Haggarty, "A Method of Side-Lobe Suppression in Phase-Coded Pulse Compression Systems," Technical Report 209, Lincoln Laboratory, M.I.T. (28 August 1959), DDC 229403.

20. P. M. Woodward, Probability and Information Theory with Applications to Radar (McGraw-Hill, New York, 1953).

21. W. R. Pratt, "Transverse Equalizer for Suppressing Distortion Echoes in Radar Receivers," Proceedings of Symposium on Pulse Compression Techniques and Applications, RADC-TDR-62-580, April 1963.

22. H. A. Wheeler, "The Interpretation of Amplitude and Phase Distortion in Terms of Paired Echoes," Proc. IRE 27, 359 (June 1939).

23. G. R. Curry, "Introduction to the TRADEX Radar System," Technical Report 357, Lincoln Laboratory, M.I.T. (15 July 1964), DDC 448350.

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13. ABSTRACT

There is a basic constraint imposed on radar measurements when the transmitted signal is a simple, CW pulse. As a result, the radar designer may wish to use a more complex signal and thereby extract more information from an echoing object. Unfortunately, the processed returns of the sophisticated waveforms usually consist of a peak response and small adjacent responses (called "time sidelobes") which degrade the resolving capability of the radar.

One means of reducing the sidelobes is to use a transversal equalizer in the radar receiver. A mathematical model of the equalizer is presented in this article and the sensitivities of an equalized system to uncompensated Doppler frequency shifts are computed. It is shown that the Doppler frequency band over which the equalizer is effective decreases as (1) the signal bandwidth decreases, (2) the distances of the large sidelobes from the main pulse increase, and (3) the magnitudes of the uncompensated sidelobes increase.

Testing the equalizer in a highly sensitive, tracking radar indicates that sidelobes 40 db below the main response can be obtained for a fixed Doppler frequency shift. Experimental data illustrating the stability of an equalized system in the presence of a Doppler shift show that, for a typical pulse-compression system, the equalizer utility is generally confined to situations where the frequency shift is within ±2 percent of the signal bandwidth.

14. KEY WORDS

radar receivers   transversal equalizers   pulse compression